1. Consider the following two systems of linear equations:

\[
\begin{align*}
5x + y - 3z &= 0 \\
-9x + 2y + 5z &= 1 \\
4x + y - 6z &= 9
\end{align*}
\quad
\begin{align*}
5x + y - 3z &= 0 \\
-9x + 2y + 5z &= 5 \\
4x + y - 6z &= 45
\end{align*}
\]

It can be shown that the first system has a solution. Use this fact to show the second system must have a solution.

2. A function \( T : \mathbb{R}^3 \to M_{2\times 2} \) is defined as follows:

\[
T \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} 0 & a - 2b \\ a - 2b & b - c \end{bmatrix}.
\]

(a) \( T \) is a linear transformation. What would you have to show to verify this fact?

(b) Which, if any, of the following vectors are in \( \ker(T) \)?

\[
\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 4 \\ 2 \\ -4 \end{bmatrix}
\]

(c) Find a basis for \( \ker(T) \).