Are the polynomials listed below linearly independent in $\mathbb{P}_2$?

$1 - 3t, \ 1 + t^2, \ 1 - 3t + t^2$. 
Let $B = \{1 + t, 2 - t\}$ and $C = \{1 - t, t\}$ be two bases for $\mathbb{P}_1$.

1. Express $2t + 2$ in $B$ and $C$ coordinates.

2. Is it easier to express the $B$ basis vectors in $C$ coordinates, or the $C$ basis vectors in $B$? If you want to find the change of coordinate matrices, why is it useful to answer this question before you begin computing anything?

3. Find the change of coordinate matrices from $B$ to $C$ coordinates and from $C$ basis vectors in $B$ coordinates.
Let $\mathcal{B} = \{b_1, b_2\}$ and $\mathcal{C} = \{c_1, c_2\}$ be bases for $\mathbb{R}^2$. Find the change-of-coordinates matrices from $\mathcal{C}$ to $\mathcal{B}$ and from $\mathcal{B}$ to $\mathcal{C}$.

\[
b_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad c_1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \quad c_2 = \begin{bmatrix} -5 \\ 2 \end{bmatrix}\]